

## Challenge 2: Binary Tree Traversal

Consider a binary tree:

```
class Tree {
    Tree left, right, parent;
    bool mark;
}
```

We are given a binary tree with the following properties:

- It is well formed, in the sense that following a child pointer (`left` or `right`) and then following a `parent` pointer brings us to the original node. Moreover, the `parent` pointer of the root is null.
- It has at least one node, and each node has 0 or 2 children.

We do not know the initial value of the `mark` fields.

Our goal is to set all `mark` fields to true. The algorithm below (Morris 1979) works in time linear in the number of nodes, as usual, but uses only a constant amount of extra space.

```
void markTree(Tree root) {
    Tree x, y;
    x = root;
    do {
        x.mark = true;
        if (x.left == null && x.right == null) {
            y = x.parent;
        } else {
            y = x.left;
            x.left = x.right;
            x.right = x.parent;
            x.parent = y;
        }
        x = y;
    } while (x != null);
}
```

**Tasks.** Prove that:

1. upon termination of the algorithm, all `mark` fields are set
2. the tree shape does not change
3. the code does not crash, and
4. the code terminates.

As a bonus, prove that the nodes are visited in depth-first order.